Fano blockade by a Bose-Einstein condensate in an optical lattice

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We study the transport of atoms across a localized Bose-Einstein condensate in a one-dimensional optical lattice. For atoms scattering off the condensate we predict total reflection as well as full transmission for certain parameter values on the basis of an exactly solvable model. The findings of analytical and numerical calculations are interpreted by a tunable Fano-like resonance and may lead to interesting applications for blocking and filtering atom beams.

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An understanding of the transport properties of ultracold atoms is vital for the development of technological applications in the fields of matter-wave interferometry [1] or quantum information processing with neutral atoms [2, 3, 4, 5]. Over the last couple of years it has been shown that optical lattices, generated by counterpropagating laser beams and providing a periodic potential modulation for the atoms, introduce many interesting and potentially useful effects by modifying single atom properties and enhancing correlations between atoms [6]. Here we discuss the scattering of atoms across a localized Bose-Einstein condensate (BEC) in an optical lattice. We find that dramatic effects of scattering resonances with either full transparency or total reflection can occur.

Previously, transparency effects have been conjectured for the scattering of He atoms on a film of superfluid helium-4 [7] and similar effects have been predicted for the scattering of atoms on a BEC in a trap of finite depth [8]. These effects were first attributed to the coherent interactions within the target and with the scattering atoms but a full understanding of the numerical results was not achieved. More numerical results were produced later [9] and a Levinson theorem was proved on general grounds [10] without revealing the mechanism for transparency effects.

In this Letter we present and analyse a very simple and analytically solvable one-dimensional model of atom scattering by a BEC. The model shows transparency as well as blockade of atoms by total reflection, which is interpreted as a Fano resonance. In the problem studied by Fano, the resonance may both enhance and suppress scattering due to interference [11]. Here the atom-atom interaction leads to an effective nonlinearity. It was shown recently [12], that nonlinearity generates several scattering channels which can lead to destructive interference and Fano resonances similar to the original Fano problem. Proposed applications in nonlinear optics and Josephson junction networks encounter various difficulties due to inhomogeneities and dissipation [13]. They are absent in the present study, thus making the resonant atom-BEC scattering ideal for harvesting on Fano resonances.

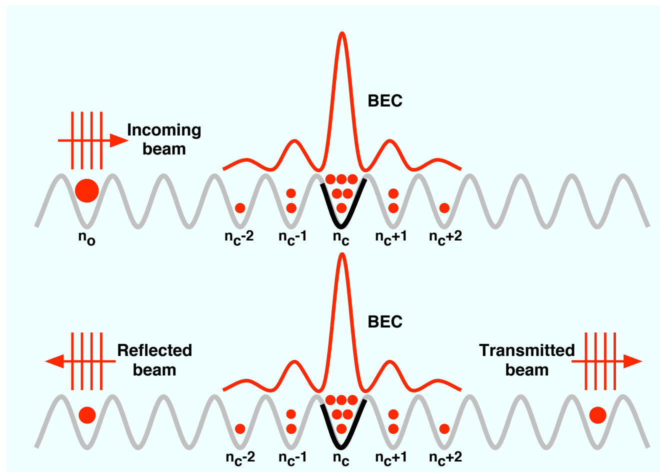

FIG. 1: (Color online). Scattering scheme in an optical lattice. The incoming, reflected, and transmitted beams of atoms are represented as plane waves. The BEC is centered in $n = n_c$, the only nonlinear lattice site.

We consider a BEC on a lattice, where interactions between atoms are present on one lattice site only (see Fig. 1). Such a situation could be realized experimentally by combining optical lattices with atom-chip technology [14, 15] or in optical micro-lense arrays [16] where the s-wave scattering length of atoms can be tuned by an inhomogeneous magnetic [17, 18] or laser field [19, 20]. Specifically we consider the discrete nonlinear Schrödinger (DNLS) equation, a classical variant of the Bose-Hubbard model appropriate for a BEC in a periodic potential in the tight binding limit [6]. With interactions being present only on site number n_c, we write in dimensionless form

$$i\frac{\partial \Psi_n}{\partial t} = -\left(\Psi_{n+1} + \Psi_{n-1}\right) - \gamma |\Psi_{n_c}|^2 \Psi_{n_c} \delta_{n,n_c}, \qquad (1)$$

where $\Psi_n(t)$ is a complex amplitude of the BEC field at site n and $-\gamma = U/J$ is the interaction strength on site n_c. This simple model reflects generic features of BECs in a one-dimensional optical lattice with inhomogeneous scattering length. Furthermore, this model could

be realized quantitatively in a deep optical lattice with tight transverse confinement [21]. For atoms with mass M in a lattice with spacing d in the tight binding limit, $J \approx 4s^{3/4}e^{-2\sqrt{2}}/\sqrt{\pi}E_r$ is the energy scale for tunneling between the lattice sites, where $s = V_0/E_r$ is the depth of the optical lattice V_0 measured in units of the recoil energy $E_r = 2\hbar\pi^2/(d^2M)$. The on-site interaction energy per atom is $U = 4\pi a_s\hbar^2\int d^3x|\psi(\mathbf{x})|^4/M$, where a_s is the tunable s-wave scattering length at the nonlinear site n_c and $\psi(\mathbf{x})$ is the localized Wannier state associated with the lowest Bloch band of the lattice. The number of atoms in the lattice is given by $N = \sum_n |\Psi_n|^2$. Small-amplitude plane-wave solutions of Eq. (1) take the form $\Psi_n = \Psi_0 \exp(ikn)\exp(-iE_k t)$ and satisfy the relation

$$E_k \equiv -2\cos k, \tag{2}$$

which defines the band of single-particle energies $E_k \in [-2, 2]$ [see Fig.2(a)]. The unit of dimensionless energy is J and the quasi-momentum k is measured in units of d^{-1}.

First, we look for localized and stationary solutions of Eq.(1), corresponding to the BEC centered in $n = n_c$. For simplicity we assume that interactions are attractive and thus $\gamma > 0$. As an ansatz, we take an exponentially localized profile: $\Psi_n(t) = b_n(t) = b_{n_c}x^{|n-n_c|}\exp(-iE_bt)$, where b_{n_c} is the condensate amplitude, $|x| < 1$, and E_b is the respective energy. Inserting this expression into (1), we obtain that

$$E_b = -\sqrt{4+g^2} \quad \text{and} \quad x = -(E_b + g)/2, \tag{3}$$

where $g \equiv \gamma b_{n_c}^2$ $(g > 0)$. Eqs. (3) correspond to solutions for localized BECs with E_b being outside of the band E_k [$E_b < -2$, see Fig.2(a)]. They are similar to bright lattice solitons pinned to the nonlinear lattice site. These localized modes exist only above a threshold $N_b = -E_b/\gamma > N_b^{th}$ [22], given by $N_b^{th} = 2/\gamma$. We assume that N_b is significantly larger than the threshold, which should be easily achieved in a possible experiment.

By controlling the number of atoms in the BEC or by tuning the nonlinear coefficient, we can easily modify the BEC energy $E_b = -\gamma N_b$ (this is equivalent to modifying the parameter g). This is one of the keys for a tunable Fano-blockade scheme. As we will show later, this energy is directly related to the energy where zero transmission of the atom beam through the BEC is observed.

We consider three different values for g as examples: $g_1 = 0.36$, $g_2 = 0.6$, and $g_3 = 0.9$. The corresponding energies [$E_{b1} = -2.03$ (box), $E_{b2} = -2.09$ (diamond), and $E_{b3} = -2.19$ (triangle)] and the profiles of the BEC states are shown in the left part of Fig. 2(b). By doing a linear stability analysis, we find that all these solutions centered in $n = n_c$ are stable. This is important for two reasons. First, experimental generation of these states should be possible. Second, the scattering of small-amplitude beams by these modes can be viewed as

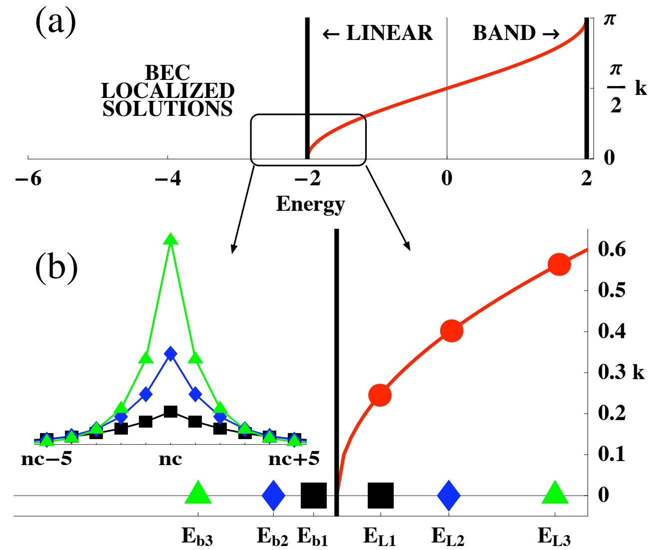

FIG. 2: (Color online). (a) Energy diagram for localized and extended solutions. In the linear band, E_k is plotted. (b) Zoom of the region $E \sim -2$. Boxes, diamonds, and triangles correspond to the BEC ($E_b < -2$) and to the local mode ($E_L > -2$) solutions for g_1, g_2, and g_3, respectively. The corresponding BEC profiles are shown. The filled circles correspond to the resonance condition (12).

a small perturbation of the latter, which will stay small if the BEC is stable, but would grow (and the BEC would decay) otherwise.

In order to study the scattering of a propagating atom beam by the localized BEC, we consider:

$$\Psi_n(t) = \phi_n(t) + b_n(t), \tag{4}$$

where $\phi_n \ll b_n$. We linearize Eq.(1) with respect to $\phi_n(t)$, obtaining the following equation for the atom beam:

$$i\frac{\partial\phi_n}{\partial t} = -(\phi_{n+1} + \phi_{n-1}) - g(2\phi_{n_c} + e^{-2iE_bt}\phi_{n_c}^*)\delta_{n,n_c}\ . \tag{5}$$

Far away from $n = n_c$, the solution of this equation corresponds to a propagating plane wave which satisfies (2). The localized BEC generates a scattering potential for the atom beam which has a constant and a time-dependent part. Equation (5) is similar to the one found in Ref.[12] for the scattering of plane waves against a discrete breather. In that work the appearance of Fano resonances [11] was studied in a full nonlinear lattice in the approximation of a very localized nonlinear mode. The scattering potential is then reduced to one lattice site and implies $g \gg 1$, which makes the discrete breather a very strong scatterer for almost all incoming plane waves. Our setup allows us to tune g to smaller values, and thus admits Fano resonances on a background of almost perfect transmission.

We solve the equation (5) by using a Bogoliubov trans-

formation [9] given by:

$$\phi_n(t) = u_n e^{-iEt} + v_n^* e^{-i(2E_b - E)t} \ , \tag{6}$$

and by inserting it in (5) we obtain the discrete Bogoliubov (DB) equations:

$$E u_n = -(u_{n+1} + u_{n-1}) - g(2u_{n_c} + v_{n_c})\delta_{n,n_c}, \tag{7}$$
$$(2E_b - E) v_n = -(v_{n+1} + v_{n-1}) - g(2v_{n_c} + u_{n_c})\delta_{n,n_c}. \tag{8}$$

Here u_n corresponds to an *open* channel which, far away from n_c, represents a propagating atom beam for which the energy is in the band $E = E_k \in [-2, 2]$. Contrary, v_n represents a *closed* channel whose extended states far away from the scattering center have $2E_b - E \notin [-2, 2]$. They are thus located outside the open channel continuum and cannot be excited in the same energy range. However, even in the absence of any coupling between both channels, the scattering center provides also a localized state in the spectrum of the closed channel with energy E_L. The localized state, by definition, is located outside the band of extended states in the v_n channel, but may be located inside the u_n-channel band $E \in [-2, 2]$. In such a case, taking the coupling between channels into account, we encounter a Fano resonance for $E_L = E_k$.

Let us consider the situation when both channels are decoupled. For this particular situation, the closed channel equation is given by

$$(2E_b - E) v_n = -(v_{n+1} + v_{n-1}) - 2g v_{n_c} \delta_{n,n_c},$$

and admits a localized solution: $v_n = v_{n_c}\ w^{|n-n_c|}$ ($|w| < 1$), for which

$$w = -g + \sqrt{1+g^2} \quad \text{and} \quad E = E_L \equiv 2(E_b + \sqrt{1+g^2}).$$

We call this solution the local mode (LM). E_L corresponds to the LM energy, which is always inside the continuum of the open channel: if $g \to 0 \Rightarrow E_L \to -2$ and, if $g \to \infty \Rightarrow E_L \to 0$. Therefore, due to the time dependence of the original scattering potential, the closed channel is able to resonate with the open one at a frequency that depends on externally controllable parameters.

Keeping in mind the localized nature of the LM and the propagating one of the open channel, we make the following ansatz:

$$u_n = \begin{cases} a_1\ e^{ik(n-n_c)} + b_1\ e^{-ik(n-n_c)} & ;\ n < n_c \\ c_1\ e^{ik(n-n_c)} & ;\ n \geq n_c \end{cases} , \tag{9}$$

$$v_n = \bar{v}_{n_c}\ \bar{w}^{|n-n_c|}. \tag{10}$$

Here, a_1, b_1 and c_1, represent the incoming, reflected and transmitted beam amplitudes, respectively. $\bar{v}_{n_c}$ correspond to the closed channel amplitude and $|\bar{w}| < 1$. The beam quasimomentum k can be generated in the experiment by using a phase imprinting method [23], Bragg scattering, or simply by acceleration of the matter-wave probe in an external potential. We solve analytically the scattering problem by inserting (9) and (10) in (7) and (8) for $n = n_c, n_c \pm 1$. By doing so, we obtain that the open channel satisfies (2), $a_1 + b_1 = c_1$, $\bar{w} = w$, and that the transmission is given by:

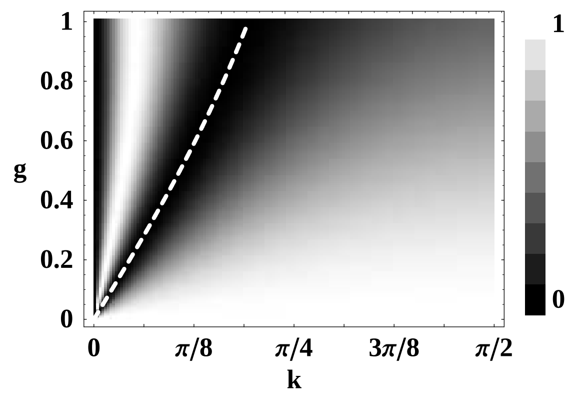

FIG. 3: T versus k and g. The dotted white line correspond to $T(k_L) = 0$, the resonance position.

$$T(k) = \frac{4\sin^2 k}{4\sin^2 k + \left(2g + \frac{g^2}{\sqrt{(E_k - 2E_b)^2 - 4} - 2g}\right)^2} \tag{11}$$

($T \equiv |c_1/a_1|^2$). Resonances occur when the denominator diverges or when $\sqrt{(E_k - 2E_b)^2 - 4} - 2g = 0$. The condition for the resonance is

$$E_k = E_L \Rightarrow k_L \equiv \arccos(-E_L/2). \tag{12}$$

Eq.(12) implies that the transmission for atom beams through the BEC is reduced to zero when a LM is generated in the process, i.e. when the closed channel resonates with the open one.

In Fig.2(b), we show the corresponding energies for the LM's, for the three values of g, $E_{L1} = -1.94$ (box), $E_{L2} = -1.84$ (diamond), and $E_{L3} = -1.69$ (triangle). The curve in that figure corresponds to Eq. (2), and the filled circles correspond to the value of k for which the resonance is expected (12). In principle, any energy in the interval $\{-2, 0\}$ can be a good candidate for the observation of a Fano resonance in this setup. But, as we will show later, the response of the system is not always the same and it essentially depends on the BEC profile.

We compute the transmission $T = T(k, g)$ and we vary the beam velocity k and the nonlinear parameter g. Fig.3 shows this sweep of parameters, where dark and bright regions represent a low ($T \to 0$) and a high ($T \to 1$) transmission response, respectively. In Fig.4(a) we show some $T(k)$ for three values of g. As g increases the width and the position (dashed white line in Fig. 3) of the resonance increase. Thus, the more localized the BEC becomes, the stronger it reflects the atom beam off resonance. As ex-

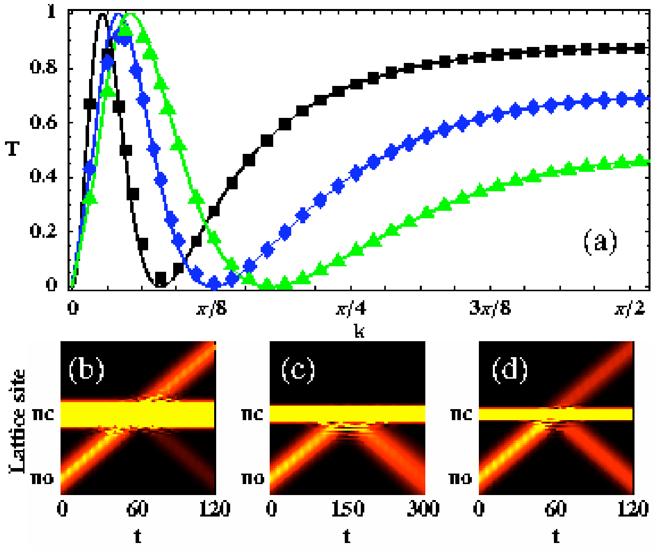

FIG. 4: (Color online). (a) T versus k for the three values of g. Lines: eq. (11), points: real time numerical simulations of eq. (1) for g_1 (boxes), g_2 (diamonds), and g_3 (triangles). (b)–(d) Evolution of $|\Psi_n(t)|^2$ in space and time: (b) $g_1, k = 1.37$, (c) $g_2, k = 0.39$, (d) $g_3, k = 1.37$.

pected from (12), increasing g (which corresponds to decreasing E_b or increasing N_b) leads to an increase of the resonance energy. By tuning the nonlinear parameter g, we can thus choose the amount of the beam which passes through the BEC. Off resonance (for larger values of k), we can select the percentage of the incoming beam that is transmitted for a defined quasimomentum. Therefore, the actual setup can be used as a 100% blockade or as a selective filter.

Now, we look for a numerical confirmation of our theoretical description for the scattering of an atom beam against a localized BEC. In the simulations, we initialize the atom beam with a Gaussian profile: $\phi = \phi_0 \exp[-\alpha(n-n_0)^2] \exp[ik(n-n_0)]$ with $\phi_0 = 0.01$ and $\alpha = 0.001$. n_0 is the initial location of the center of the distribution and well separated from the BEC to avoid an initial interaction. k is the initial quasimomentum. The amplitude ϕ_0 was chosen to be very small compared to the BEC amplitude ($\phi_0/b_{n_c} \sim 1\%$) in order to justify (5). The value of α implies a spatial width of approximately 60 sites and a reciprocal width in k-space of 0.12. With this choice we can clearly observe the resonant response of the system. In Fig.4(a) the symbols denote our numerical results for three different values of g. The agreement between theory and simulations is almost perfect. We have some disagreement for small values of k where the group velocity is too small and the computation of T is more complicated. Figs.4 (b), (c), and (d) show some numerical examples of the scattering process with a transmission of 80%, 0% and 40%, respectively. It is worth mentioning that, if we extend the nonlinearity to three sites of the array, similar transmission curves are observed. This shows the robustness of our theoretical results in a more realistic experimental setup.

In conclusion, we have investigated Fano resonances in the context of Bose-Einstein condensates in an optical lattice. The implementation of this idea can be viewed as a powerful tool for controlling the transmission of matter waves in interferometry and quantum information processes. Fano resonances rely on destructive interference and are thus inherent to wave dynamics. An observation of these resonances in atom-BEC scattering would provide, in addition to tunable filters, a new demonstration of the quantum matter wave character of ultracold atoms.

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